SLIATE

SRI LANKA INSTITUTE OF ADVANCED TECHNOLOGICAL EDUCATION

(Established in the Ministry of Higher Education, vide in Act No. 29 of 1995)

Higher National Diploma in Engineering (Electrical/Mechanical/Building Services)
Second Year, Second Semester Examination – 2016
MA 2204 / BSE 2207 – Advanced Engineering Mathematics

Instructions for Candidates:	No. of questions : 5
Answer any **four (4)** questions.	No. of pages : 2
All questions carry equal marks.	Time : two (2) hours

1. i). Find the function of z of $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ at the conditions of $\frac{\partial z}{\partial y} = -2\sin y$ and z = 0 when x = 0.

[15 marks]

ii). Consider the wave equation given below.

$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, where c is a constant.

Show that the one solution of this equation can be given as y(x,t) = cos t. sin x.

[10 marks]

[Total 25 marks]

2. f(x) is a function of period 2π such that f(x) = x+1 for $0<x<\pi$.

i). Sketch the graph of f(x) according to the cosine series in the interval of $-3\pi<x<3\pi$.

[5 marks]

ii). Find the Fourier cosine series for the function of f(x).

[10 marks]

iii). By substituting suitable values of x, prove that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

[10 marks]

[Total 25 marks]

3. a). Find the Laplace transforms of the following.

i). $3t^2 - \cos 2t$

ii). $e^{-2t} \sin 2t \cos 3t$

iii). $t \cos 3t$ [15 marks]

b). Find the inverse Laplace transforms of the following.

i). $\frac{2s^2-3s+4}{s(s-1)(s-2)}$

ii). $\frac{1}{(s+2)^4}$

[10 marks]

[Total Marks 25]

4. a). Solve the following differential equation by using Laplace transform method.

$2y'' + 5y' + 2y = e^{-2t}$, initial conditions are y (0)=1 and y'(0)=1. [10 marks]

b). Find the Z transform of the following.

i). $\frac{1}{2}^k$, where k≥0

ii). 3k+2, where $-1 \geq k \geq 3$

[10 marks]

c). Find the inverse Z transform of $\frac{1}{(z-3)(z-2)}$, where $|Z| < 2$

[05 marks]

[Total Marks 25]

5. Bessel function is given as $J_n(x) = \frac{x^n}{2} \sum_{k=0}^{\infty} \frac{(-x^2/4)^k}{k!(k+n)!}$.

i). Find $J_2(2)$, $J_1'(2)$, $J_2'(2)$ in terms of a and b.

where $J_0(2) = a \; and \, J_1(2) = b$ [15 marks]

ii). Prove that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$. [10 marks]

[Total Marks 25]

Table I Standard Laplace-Transforms

$f(t)$	$\mathcal{L}\{f(t)\}$	$f(t)$	$\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
k	$\frac{k}{s}$	$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$
e^{at}	$\frac{1}{s-a}$	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$	$e^{at}\sinh \omega t$	$\frac{\omega}{(s-a)^2-\omega^2}$
t	$\frac{1}{s^2}$	$e^{at}\cosh \omega t$	$\frac{s-a}{(s-a)^2-\omega^2}$
t^2	$\frac{2!}{s^3}$	$y(t)$	$Y(s)$
t^n	$\frac{n!}{s^{n+1}}$	$\frac{dy}{dt}$	$sY(s)-y(0)$
$e^{at}t^n$	$\frac{n!}{(s-a)^{n+1}}$	$\frac{d^2y}{dt^2}$	$s^2Y(s)-sy(0)-y'(0)$